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$$\frac{\int \cdots \int_T dx_1 \cdots dx_n}{\int \cdots \int_S dx_1 \cdots dx_n};$$

in other words, find the magnitude of a "solid angle" in n dimensions, with the "sphere" as unit solid angle.

Note. This problem was discussed and left unsolved by Schläfli in the *Quarterly Journal of Mathematics* for 1858, 1860, 1867.—Editor.

349 (Calculus) [December, 1913]. Proposed by C. N. SCHMALL, New York City.

If $y = a \cos (\log x) + b \sin (\log x)$, eliminate the constants a and b and obtain the equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0.$$

198 (Number Theory) [November, 1913]. Proposed by the late ARTEMAS MARTIN.

Prove that every even number is the sum of two prime numbers.

Note. This problem has long been known and no proof has ever been given.—Editor.

201 (Number Theory) [December, 1913]. Proposed by E. T. BELL, University of Washington.

Eisenstein proposed (*Crelle*, t. 27, p. 282), as the simplest of several problems: "In the expansion of

$$\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1 - z)^{p-1}} - 1,$$

where p is prime, to show that the coefficients of the various powers of z are all divisible by p ."

SOLUTIONS OF PROBLEMS.

231 (Number Theory) [April, 1915]. Proposed by A. J. KEMPNER, University of Illinois.

Is the series whose terms are the reciprocals of all positive integers not containing a given combination of figures, for example not containing the combination 37, convergent or divergent? Numbers such as $\frac{1}{37}$, $\frac{1}{370}$, $\frac{1}{3371}$ shall be omitted, numbers such as $\frac{1}{73}$, $\frac{1}{307}$, $\frac{1}{3317}$ shall be admitted as terms of the series. (Compare AMERICAN MATHEMATICAL MONTHLY, Volume 21, page 123.)

SOLUTION BY FRANK IRWIN, University of California.

Let $f(n)$ be the number of numbers in the class, g_n , of numbers of n digits that do not contain the given combination, for example 37. We can get these numbers by taking any number of g_{n-1} and adding a digit at the end; except that this digit must not be 7 if the number chosen from g_{n-1} ends with a 3. Since the number of these rejected numbers of g_{n-1} is evidently $f(n-2)$, we have the formula: $f(n) = 10f(n-1) - f(n-2)$. In the general case, where the given combination consists of k digits, we should get similarly, for $n > k$,

$$f(n) = 10f(n-1) - f(n-k). \quad (1)$$

(An exception would arise for such a combination as 37537; here, for instance, the formula $f(8) = 10f(7) - f(3)$ would not hold, for the number 375 belonging to g_3 would have been subtracted, as it should not have been, since the number obtained by writing after it 3753, viz., 3753753, does not belong to g_7 , and so has not been counted in $f(7)$. It will be seen that such cases arise when the first l digits of the given combination are the same as the last l , $l < k$. We shall call such combinations "improper" and deal with them separately.)

From (1) we have $f(n-1) > f(n)/10$. Also since $f(n-1) = f(n)/10$, $n < k$, and $f(n-1) \geq f(n)/10$, $n = k$ (= if the given combination begins with a zero), we have, for $n > k$, the string of relations $f(n-2) \geq f(n-1)/10$, $f(n-3) \geq f(n-2)/10$, $\dots$. From these last inequalities we derive $f(n-k) \geq f(n-1)/10^{k-1}$. Comparing this with (1), we see that $f(n) \leq 10f(n-1) - f(n-1)/10^{k-1}$, or $f(n) \leq 10rf(n-1)$, if we put $(10^k - 1)/10^k = r$. Since the reciprocal of any number in g_n is $\leq 1/10^{n-1}$, it follows that the sum of the reciprocals of the numbers in $g_k, g_{k+1}, \dots$ are less than the corresponding terms of the series

$$\frac{f(k)}{10^{k-1}} (1 + r + r^2 + \dots).$$

This series converges, for r is < 1 ; and hence our given series converges.

The case of "improper" combinations may be rapidly dealt with by the following device. Write the digit a , k times at the end of the given combination, where a is any digit other than the first digit of the combination. This certainly gives us a "proper" combination (of $2k$ digits), so that, as already proved, the series formed from the harmonic series by omitting terms whose denominators contain this combination converges. But this series evidently contains all terms of the given series; this latter, then, converges too.

In like manner we may prove that *the series obtained by omitting terms containing all of a given set of combinations (some of which may be repeated) converges*. Here we may compare the series with that obtained by omitting terms containing the single combination formed from the given combinations by juxtaposition.

This supplies another proof of the proposition in my paper on "A Curious Convergent Series" in this MONTHLY for May, 1916, that proposition being the special case of the above in which each of the combinations consists of a single digit.

The proposition stated by Professor Moulton in his solution of Algebra 453 in this MONTHLY for Oct., 1916, refers, I suppose—the proof is not given in full—not to combinations of the kind here considered, but to selections, and without any reference to the juxtaposition of the digits of the selection in the omitted terms.

239 (Number Theory) [March, 1916; March, 1919]. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

Give a general method for determining the solution in integers of the equation

$$x^r - 10xy - (n + 1) + y = 0,$$

where r and n are positive integers.

II. SOLUTION BY FRANK IRWIN, University of California.

If $x = 0$, $y = n + 1$; noting this solution, we shall suppose in what follows $x \neq 0$.

The degree in x of the given equation may be reduced by the following device. $y - (n + 1)$ must be divisible by x , say $y - (n + 1) = y_1x$. Then our equation reduces to

$$x^{r-1} - 10xy_1 - 10(n + 1) + y_1 = 0.$$

Putting $y_1 - 10(n + 1) = y_2x$, $y_2 - 10^2(n + 1) = y_3x$, etc., we finally reach the equation

$$1 - 10xy_r - 10^r(n + 1) + y_r = 0, \quad (1)$$

where y and y_r are connected, it will be found, by the relation

$$y = (n + 1)[1 + 10x + 10^2x^2 + \dots + 10^{r-1}x^{r-1}] + y_rx^r,$$

or

$$y = (n + 1) \frac{10^rx^r - 1}{10x - 1} + y_rx^r. \quad (2)$$

Rewriting (1) as

$$(10x - 1)y_r = 1 - 10^r(n + 1), \quad (3)$$

we see that x and y_r must have opposite signs.

(i) Let y_r be negative, $y_r = -y'_r$. Then (3) becomes $(10x - 1)y'_r = 10^r(n + 1) - 1$. Here the number on the right ends with the digit 9, as does also the first factor on the left. Hence we see that y'_r must end with 1. We get, then, a solution by taking for y'_r any factor, ending with 1, of the right side, and determining x and y from (3) and (2).

(ii) Let x be negative, $x = -x'$. Then (3) becomes

$$(10x' + 1)y_r = 10^r(n + 1) - 1.$$

We take y_r equal to any factor of the right side that ends with 9.

This gives us all solutions.

Suppose that $10^r(n + 1) - 1$ can be factored as $a \cdot b$, where a ends with 1, b with 9. Then